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Indirect interaction of colloidal particles adsorbed on smectic films

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Colloidal particles like peptides and proteins adsorbed on a stack of lipid bilayers cause elastic deformations which disturb the smectic order. Two adsorbed particles attract each other due to the superposition of their deformation fields. The effective pair potential attributed to this substrate-mediated force decays exponentially with the particle distance. The range of this potential coincides with the decay length of elastic deformations, and is found to be proportional to the square root of the stack thickness. If the stack is sufficiently thick, the substrate-mediated interaction is estimated to be strong enough to overcome the entropic barrier and enforce aggregation or even crystallization of the adsorbate.

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I. INTRODUCTION

A cell membrane is a lipid bilayer which contains a variety of polymeric inclusions such as integral proteins [1]. In addition to the embedded transmembrane particles, there are also amphipathic peptides and proteins adsorbed on the membrane surface. The lateral ordering of these molecules can influence their biological activity. There exist substantial interactions between protein molecules embedded in biological membranes. In protein-rich systems, the interplay of various intermolecular forces can result in phase separations as well as in two-dimensional crystallization [2]. A practical situation in which these interactions are important is the crystallization of membrane proteins for electron microscopy and x-ray diffraction studies [3].

Theoretical [4] and experimental investigations [5] revealed that embedded proteins interact indirectly via deformations of the lipid bilayer. Each inclusion is the source of elastic membrane deformations, which extend over distances substantially larger than the cross section diameter of a lipid molecule. Superposition of the deformation fields produces indirect forces between the macromolecules. The strength of these forces depends on the elastic properties of the lipid bilayer. Elastic deformations have also been suggested to give rise to attractive forces between parallel DNA molecules adsorbed on cationic lipid bilayers [6,7]. Furthermore, indirect forces between inclusions, which are incorporated into the bulk of a smectic liquid crystal, should also occur. Comprehensive theoretical studies of Turner and co-workers [8–11] dealt with the interaction between inclusions residing in the interior of lamellar phases consisting of regularly spaced lipid bilayers or diblock copolymers. The effective pair potential for two inclusions was found to be attractive on average, and strongly anisotropic. Rather complicated effective pair potentials result if higher order terms of a multipole expansion for the interaction potential are taken into account [10].

Electron microscopy [12], scanning tunneling [13], and atomic force microscopy [14,6] offer excellent possibilities to study lipid bilayers and multilayers. The familiar Langmuir-Blodgett technology is suitable to transfer stacks with a few or even several dozen lipid layers to a plane solid support. We suppose that proteins or peptides are adsorbed on the lipid-water interface of such films. Lipid-mediated interactions are expected to be dependent on the number of layers in the smectic film. Changing the number of bilayers in Langmuir-Blodgett films could be a possible means to modify the order of adsorbates consisting of colloidal molecules.

In this paper we investigate how the stack thickness influences the indirect interaction between the colloidal particles. In addition to stacks of a moderate thickness, very thin films only consisting of one bilayer resting on a solid support are also considered. The results for very thin films and stacks differ in the interaction range. It will be shown that the range of the membrane-mediated attraction force grows with an increasing number of bilayers in a stack. The effective interaction radius is found to be proportional to the square root of the stack thickness. In sufficiently thick films the interaction range could be large enough to overcome the entropic barrier and enforce aggregation or even crystallization of the adsorbate. Provided that the distances between the particles are much larger than their diameters, the theoretical approach is also applicable if the upper bilayer of the Langmuir-Blodgett film contains membrane-spanning proteins (Fig. 1).

II. ELASTIC DEFORMATIONS OF A VERY THIN SUPPORTED FILM

Let us consider a large colloidal particle adsorbed on the surface of a very thin soft film which rests on a solid support. The film thickness d_0 is assumed to be comparable to or even substantially smaller than the cross-sectional diameter

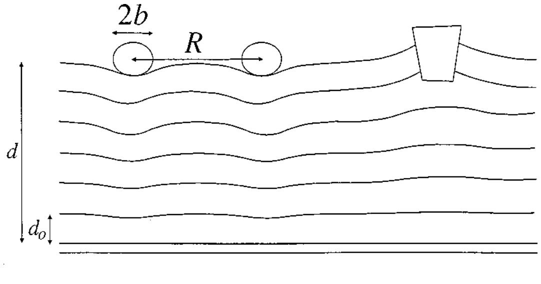

FIG. 1. Colloidal molecules adsorbed on a stack of bilayers produce a disturbance of the flat interface profile. Integral proteins spanning the upper bilayer have the same effect.

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of the adsorbed particle. Forces which attach the colloid to the substrate are the source of elastic deformations accompanied by the change of the film surface profile. Deviations u of the film thickness from the unperturbed flat state are expressed as a function of Cartesian coordinates $X=(x,y)$, which are introduced in such a way that a point of the film surface is defined by the position vector $[x,y,z=d_0+u(x,y)]$. The free elastic energy F of a thin film should be a functional of the surface profile $u(X)$. If film deformations are small $[|u(X)|\ll d_0]$, a functional Taylor expansion about the flat configuration $[u(X)=0]$ is justified. Neglecting higher order terms, we obtain

$$F[u]=F[u=0]+\int C_u(X)u(X)d^2X+\tfrac{1}{2}\int\int C_{uu}(|X-\overline{X}|)u(X)u(\overline{X})d^2X\,d^2\overline{X}+\cdots,\tag{1}$$

where the notation for functional derivatives,

$$C_u(X)=\left(\frac{\delta F}{\delta u(X)}\right)_0$$

and

$$C_{uu}(|X-\overline{X}|)=\left(\frac{\delta^2 F}{\delta u(X)\,\delta u(\overline{X})}\right)_0,$$

is used. In the force-free state the flat configuration $u(X)=0$ is stable, and thus the first derivative $C_u(X)$ must be equal to zero. If the film is subjected to an external load, $C_u(X)\equiv -f(X)$ is the stress normal to the film surface. For deformations with small wave numbers, $q\ll 2\pi/d_0$, a gradient expansion is allowed. Substituting $\overline{X}$ with $S=\overline{X}-X$, and expanding about $S=0$, the second integral in Eq. (1) is transformed into

$$\int\int C_{uu}(|S|)u(X)u(X+S)d^2X\,d^2\overline{X}=\int u(X)[B'u(X)-\sigma'\nabla^2u(X)+K'\nabla^4u(X)]d^2X,\tag{2}$$

where gradient terms with higher order than four are neglected, and $\Delta=\nabla^2=\partial_x^2+\partial_y^2$ denotes the Laplace operator in two dimensions. The coefficients in expansion (2) are formally defined as

$$B'=\int_0^\infty C_{uu}(s)2\pi s\,ds,$$

$$\sigma'=-\frac{1}{4}\int_0^\infty s^2C_{uu}(s)2\pi s\,ds$$

and

$$K'=-\frac{1}{64}\int_0^\infty s^4C_{uu}(s)2\pi s\,ds,$$

with $s=|S|=\sqrt{S_1^2+S_2^2}$. Performing several partial integrations, the elastic free energy (1) can be cast into the form

$$F[u]=\tfrac{1}{2}\int[B'(u(X))^2+\sigma'(\nabla u(X))^2+K'(\Delta u(X))^2-2f(X)u(X)]d^2X.\tag{3}$$

Equation (3) has the same mathematical structure as a well-known free energy expression used in the elastic continuum theory of lipid membranes [15]. B' is the compression-expansion modulus, σ' the surface tension, and K' the bending modulus. The stability of the flat film geometry in the force-free state requires $B'>0$, $\sigma'\geqslant 0$ and $K'>0$. The corresponding Eulerian equation

$$K'\Delta\Delta u(X)-\sigma'\Delta u(X)+B'u(X)=f(X)\tag{4}$$

is a condition necessary for a minimum of the free energy. It is useful to evaluate the Green's function by solving the equation

$$K'\Delta\Delta G(|X-\overline{X}|)-\sigma'\Delta G(|X-\overline{X}|)+B'G(|X-\overline{X}|)=\delta(X-\overline{X}),\tag{5}$$

where $\delta(X-\overline{X})$ is the Dirac δ function. $G(|X-\overline{X}|)$ describes the film response to a force applied at the point $\overline{X}$. Then the interface profile for any force distribution is obtained from the convolution integral

$$u(X)=\int G(|X-\overline{X}|)f(\overline{X})d^2\overline{X}.\tag{6}$$

The Green's function is evaluated by applying the two-dimensional Fourier transform. Inserting

$$G(x-\overline{x},y-\overline{y})=\frac{1}{(2\pi)^2}\int\int\tilde{G}(q_x,q_y)\exp[iq_x(x-\overline{x})+iq_y(y-\overline{y})]dq_xdq_y,$$

and the corresponding ansatz for the Dirac δ function into Eq. (5), we arrive at $[K'q^4+\sigma'q^2+B']\tilde{G}(q)=1$, where $q=\sqrt{q_x^2+q_y^2}$. If $\sigma'>0$ the term Kq^4 can be neglected when large distances or small wave numbers $q\ll\sqrt{\sigma'/K}$ are considered, and thus the simpler equation $[\sigma'q^2+B]\tilde{G}_I(q)=1$ results. The inverse Fourier transform of $\tilde{G}_I(q)$ can be written as a Hankel transform, namely,

$$G_I(R)=\frac{1}{2\pi B'}\int_0^\infty\frac{J_0(qR)q\,dq}{1+\xi_I^2q^2},\tag{7}$$

where the notation $R=\sqrt{(x-\overline{x})^2+(y-\overline{y})^2}$ and $\xi_I=\sqrt{\sigma'/B'}$ is used, and $J_0(qR)$ is the Bessel function of the first kind. The formula $2J_0(qR)=H_0^{(1)}(qR)+H_0^{(2)}(qR)$ connects the Bessel function to Hankel functions of first and second orders, and taking into account the relation $H_0^{(2)}(qR) = -H_0^{(1)}(-qR)$, which is valid for real values of the argument qR, we obtain

$$J_0(qR) = \frac{H_0^{(1)}(qR) - H_0^{(1)}(-qR)}{2}. \tag{8}$$

Inserting Eq. (8) into Eq. (7) yields the integral

$$G_I^S(R) = \frac{1}{4\pi B'} \int_{-\infty}^{\infty} \frac{H_0^{(1)}(qR)q\,dq}{1+\xi_I^2 q^2}, \tag{9}$$

which can be replaced by the sum of the residues attributed to the poles located in the upper half of the plane of complex numbers $q = \mathrm{Re}\,q + i\,\mathrm{Im}\,q$. The denominator in integral (9) has the zero $q = i\xi_I^{-1}$ located in the upper half-plane. Thus we obtain the result

$$G_I^S(R) = \frac{2\pi i}{4\pi B'} \frac{H_0^{(1)}(iR/\xi_I)}{2\xi_I^2},$$

which is simplified to

$$G_I^S(R) = \frac{1}{2\pi B' \xi_I^2} K_0\left(\frac{R}{\xi_I}\right) \tag{10}$$

by using the relation $\frac{1}{2}\pi i H_0^{(1)}(iR/\xi_I) = K_0(R/\xi_I)$, where K_0 is the modified Bessel function [16]. If R is comparable to or larger than ξ_I the formula

$$G_I^S(R) = \frac{1}{2\sqrt{2\pi}B'\xi_I^2}\left(\frac{\xi_I}{R}\right)^{1/2} \exp\left(-\frac{R}{\xi_I}\right) \tag{11}$$

is applicable. It is often assumed that the surface tension σ' of lipid bilayers vanishes. The Green's function is modified if the interface tension σ' is very small or zero, because in this case the term Kq^4 must be taken into account. Then Eq. (9) is replaced by

$$G_{II}^S(R) = \frac{1}{2B'} \int_{-\infty}^{\infty} \frac{H_0^{(1)}(qR)q\,dq}{1+\xi_{II}^4 q^4},$$

with $\xi_{II} = (K'/B')^{1/4}$, and again applying the residue method we arrive at

$$G_{II}^S(R) = \frac{2\pi i}{4\pi B'}\left[\frac{i}{4\xi_{II}^2} H_0^{(1)}\left(\frac{i\beta R}{\xi_{II}}\right) - \frac{i}{4\xi_{II}^2} H_0^{(1)}\left(\frac{i\beta^* R}{\xi_{II}}\right)\right],$$

where $\beta = (1+i)/\sqrt{2}$, and β^* is the complex conjugate of β. This expression is transformed into

$$G_{II}^S(R) = -\frac{1}{2\pi B' \xi_{II}^2} \mathrm{Im}\, K_0\left(\frac{\beta R}{\xi_{II}}\right). \tag{12}$$

Equation (12) was already derived by Boulbitch [19] for describing the deflection of a biological cell membrane under application of a local force. If the distance R is not too small compared to ξ_{II} the formula

$$G_{II}^S(R) = \frac{1}{4\sqrt{\pi}B'\xi_{II}^2}\left(\frac{\xi_{II}}{R}\right)^{1/2} h\left(\frac{R}{\xi_{II}\sqrt{2}}\right) \exp\left(-\frac{R}{\xi_{II}\sqrt{2}}\right) \tag{13}$$

is applicable, where the trigonometric function h is defined by

$$h(x) = (\sqrt{1-1/\sqrt{2}})\cos(x) + (\sqrt{1+1/\sqrt{2}})\sin(x). \tag{14}$$

Finally, we consider the case where a long rodlike molecule is adsorbed on the film surface. The Green's function for a long rod with an orientation parallel to the y axis is obtained from the integral

$$G_I^R(R) = \frac{1}{2\pi B'} \int_{-\infty}^{\infty} \frac{\exp(iq_x x)\,dq_x}{1+\xi_I^2 q_x^2},$$

which leads to the exponential function

$$G_I^R(|x|) = \frac{1}{2B'\xi_I} \exp\left(-\frac{|x|}{\xi_I}\right). \tag{15}$$

Similarly, in the case of a negligible surface tension $\sigma' \ll B'\xi_I^2$, we obtain

$$G_{II}^R(|x|) = \frac{1}{2\sqrt{2}B'\xi_{II}}\left[\cos\left(\frac{|x|}{\xi_{II}\sqrt{2}}\right) + \sin\left(\frac{|x|}{\xi_{II}\sqrt{2}}\right)\right] \times \exp\left(-\frac{|x|}{\xi_{II}\sqrt{2}}\right). \tag{16}$$

Obviously, the validity of Eqs. (10), (12), (15), and (16) is not restricted to smectic bilayer films. Since expansions (1) and (2) are applicable to any thin soft films, the results should also be valid for other materials, e.g., thin films of soft solids, if small wave number deformations ($q \ll 2\pi/d_0$) are considered.

III. COLLOIDAL PARTICLES ADSORBED ON A STACK OF BILAYERS

A. Deformation of a bilayer stack

Let us consider a stack of smectic layers or bilayers resting on a solid surface identical to the plane $(x, y, z = -d)$ of a Cartesian coordinate system. Now the shift u of smectic bilayers depends on the additional coordinate z. Colloidal particles adsorbed on the surface $z = 0$ exert a force which is the source of elastic deformations (Fig. 1). The load $f(x,y)$ is nonzero only in the small regions where colloidal particles touch the surface. The elastic free energy of the stack can be written as [20]

$$F = \frac{1}{2}\int_V d^3X\left[B\left(\frac{\partial u}{\partial z}\right)^2 + K(\nabla^2 u)^2\right] + \frac{1}{2}\int_A d^2X\,\sigma(\nabla u)^2 - \int_A d^2X\,fu, \tag{17}$$

where V is the volume, A the surface area, and the coefficients B, K, and σ are the compression modulus, bending modulus, and surface tension, respectively. We assume that the thickness d of the stack is small compared to its lateral extension. Variation of the free energy [Eq. (17)] leads straightforwardly to the Eulerian equation

$$B\frac{\partial^2 u}{\partial z^2}=K\Delta^2 u, \tag{18}$$

with boundary conditions for the adsorption plane $z=0$ and the lower plane $z=-d$, where the multilayered stack rests on the solid support:

$$B\left(\frac{\partial u}{\partial z}\right)_{z=0}-\sigma\Delta u(z=0)=f(x,y), \tag{19}$$

$$u(z=-d)=0. \tag{20}$$

Performing some partial integrations and taking Eq. (20) into account, the free energy (17) is transformed into

$$F=-\frac{1}{2}\int_V d^3X\,u\left[B\frac{\partial^2 u}{\partial z^2}-K\Delta^2 u\right]+\frac{1}{2}\int_A d^2X\,u\left[B\left(\frac{\partial u}{\partial z}\right)_{z=0}-\sigma\Delta u(z=0)\right]-\int_A d^2X\,fu,$$

and using Eqs. (18) and (19), we arrive at the simple expression

$$F=-\tfrac{1}{2}\int d^2X\,fu. \tag{21}$$

To characterize the elastic response it is sufficient to solve the Eulerian equation (18) for the Green's function $u=G(x-\overline{x},y-\overline{y},z)$, with boundary conditions

$$B\left(\frac{\partial G}{\partial z}\right)_{z=0}-\sigma\Delta G(z=0)=\delta(x-\overline{x})\,\delta(y-\overline{y}),$$

$$G(x-\overline{x},y-\overline{y},z=-d)=0.$$

Inserting the Fourier transform

$$G(x-\overline{x},y-\overline{y},z)=\frac{1}{(2\pi)^2}\int\int G_q(z)\exp[iq_x(x-\overline{x})+iq_y(y-\overline{y})]dq_x dq_y$$

into Eq. (18) yields

$$B\frac{d^2G_q(z)}{dz^2}-Kq^4G_q(z)=0, \tag{22}$$

where $q^2=(q_x^2+q_y^2)$. The boundary conditions for the function $G_q(z)$ are

$$B\left(\frac{dG_q(z)}{dz}\right)_{z=0}+\sigma q^2G_q(0)=1 \quad\text{and}\quad G_q(z=-d)=0$$

Obviously, the solution of Eq. (22),

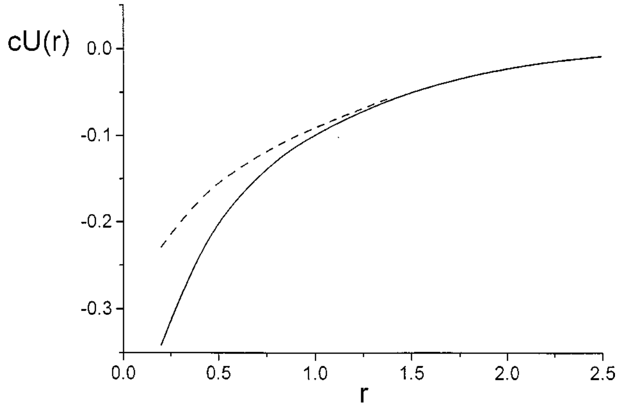

FIG. 2. Plot of the substrate-mediated pair potential (in dimensionless units) cU with $c=B\xi/P^2$, vs the reduced distance $r=R/\sqrt{\xi d}$. The solid line results from the complete Green's function (23), while the dashed line is obtained from the approximate formula (24).

$$G_q(z)=\frac{\sinh[q^2\xi(d+z)]}{q^2[\xi B\cosh(q^2\xi d)+\sigma\sinh(q^2\xi d)]},$$

with $\xi=\sqrt{K/B}$, satisfies the required boundary conditions. If the condition $\sigma/(\xi B)\ll 1$ holds, the surface tension is not important. Then the Green's function

$$G(R)=-\frac{1}{\pi^2B\xi}\sum_{n=0}^{\infty}\frac{1}{n+\frac{1}{2}}\operatorname{Im}K_0\left(\frac{\beta\sqrt{(n+\frac{1}{2})\pi}}{\sqrt{\xi d}}R\right) \tag{23}$$

is obtained for $z=0$ (see Appendix A), where $R=\sqrt{(x-\overline{x})^2+(y-\overline{y})^2}$ and $\beta=(1+i)/\sqrt{2}$. For $R>\sqrt{\xi d}$ only the first term of sum (23) has a substantial contribution (compare Fig. 2). Taking into account the asymptotic behavior of the Bessel function K_0 [16] for $R\to\infty$, one obtains the formula

$$G(r)=\frac{(2\pi)^{1/4}}{\pi^2B\xi}h\left(\frac{1}{2}\sqrt{\pi r}\right)\left(\frac{1}{r}\right)^{1/2}\exp\left(-\frac{1}{2}\sqrt{\pi r}\right), \tag{24}$$

where the scaled argument $r=R/\sqrt{\xi d}$ is used, and the function h is defined by Eq. (14). The decay length $\sqrt{\xi d}$ for perturbations of the flat surface profile increases with increasing film thickness d.

B. Substrate-mediated interaction

The Green's function allows one to determine an effective interaction potential for substrate-mediated forces between adsorbed molecules. Combining the relation

$$u(X)=\int G(|X-\overline{X}|)f(\overline{X})d^2\overline{X}$$

with Eq. (21) yields the simple formula

$$F=-\tfrac{1}{2}\int\int G(|X-\overline{X}|)f(X)f(\overline{X})d^2X\,d^2\overline{X}, \tag{25}$$

which is also applicable to very thin fims considered in Sec. III A. Let us consider two colloidal particles with coordinates X_1 and X_2. Both particles produce a stress normal to the surface, which is different from zero in the regions where the molecules touch the interface, say within the regions $|X-X_1|<a_1$ and $|X-X_2|<a_2$. Thus we can write $f(X)=P_1(|X-X_1|)+P_2(|X-X_2|)$, where the functions $P_i(|X|)$ differ from zero only within small circular regions $|X|<a_i$ ($i=1$ and 2). Inserting this expression for $f(X)$ into Eq. (25) leads to

$$F=\tfrac{1}{2}U(X_1,\overline{X}_1)+\tfrac{1}{2}U(X_2,\overline{X}_2)+U(X_1,\overline{X}_2), \tag{26}$$

where

$$U(X_i,\overline{X}_j)=-\int\int G(|X_i-\overline{X}_j|)P_i(|X_i|)P_j(|\overline{X}_j|)d^2X_i\,d^2\overline{X}_j \tag{27}$$

$(i,j=1,2)$. We consider the case of a sufficiently large distance $|\overline{X}_2-X_1|\gg a_1,a_2$ between the adsorbed molecules and fix their position vectors X_1 and $\overline{X}_2$. Then the function $G((X_i+\Delta X_i)-(\overline{X}_j+\Delta\overline{X}_j))$ of the arguments ΔX_i and $\Delta\overline{X}_j$ $(i,j=1,2)$ does not noticeably vary if $|\Delta X_1|\leqslant a_1$ and $|\Delta\overline{X}_2|\leqslant a_2$. The self-energies $\tfrac{1}{2}U(X_1,\overline{X}_1)$ and $\tfrac{1}{2}U(X_2,\overline{X}_2)$ in Eq. (26) can be omitted when the interaction energy of adsorbed molecules is evaluated. Using the notation $U(X_1,\overline{X}_2)\equiv U(R)$ with $R=|\overline{X}_2-X_1|$ the effective interaction potential

$$U(R)=-P_1P_2G(R), \tag{28}$$

with

$$P_i=\int_0^{a_i}P_i(s)2\pi s\,ds \quad (i=1,2) \tag{29}$$

is obtained from Eq. (27). Equation (28) can be considered as the leading term of a multipole expansion analogous to a corresponding expansion for inclusions localized in the bulk [10]. This term accurately describes the interaction if $R\gg a_i$ ($i=1$ and 2), and if the adsorption force P_i is substantially larger than correction terms associated with moments of the force distribution $P_i(s)$. The second condition can be written as

$$|P_i|(\sqrt{\xi d})^n\gg\left|\int_0^{a_i}s^nP_i(s)2\pi s\,ds\right| \quad (n=2,4,6,\dots)$$

for a substrate consisting of smectic layers [21].

In a similar way the interaction energy for two parallel rods with length L is found by introducing coupling constants $\hat{P}_i$ ($i=1$ and 2), which are equal to the forces per unit rod length normal to the interface. It is supposed that the mutual distance $|x|$ between parallel rods is small compared to the rod length L. In this case the potential $\hat{U}(|x|)=-\hat{P}_1\hat{P}_2G(|x|)$ is defined as the interaction energy per unit rod length, and thus the total interaction energy of two parallel rods with length L is given by $U(|x|)=L\hat{U}(|x|)$.

Finally, the case of an infinitely thick smectic stack, which fills the half-space $-\infty<z\leqslant 0$, is discussed. The Green's function for rods $G(|x-\overline{x}|)=\tfrac{1}{2}|x-\overline{x}|/(\xi B+\sigma)$, evaluated in Appendix B, does not decay for large distances $(|x-\overline{x}|\to\infty)$. The same problem arises for the corresponding Green's function attributed to a point force exerted to the surface of an infinitely thick stack. This behavior is not surprising, since even thermodynamic fluctuations destroy the one-dimensional order of an infinitely extended lamellar phase [20].

C. Statistics for an ensemble of adsorbed particles

We consider a two-dimensional system of N hard particles with interaction potential

$$W(X_{1,\dots,}X_N)=\sum_{1\leqslant i<j\leqslant N}w(|X_i-X_j|),$$

where the pair potential $w(|X_i-X_j|)=w^{\mathrm{hp}}(|X_i-X_j|)+U(|X_i-X_j|)$ consists of a hard-particle contribution and the attractive term defined by Eq. (28). In the case of disklike molecules, the hard-particle contribution is defined by $w^{\mathrm{hp}}(|X_i-X_j|)=\infty$ for $|X_i-X_j|<2b$ and $w^{\mathrm{hp}}(|X_i-X_j|)=0$ for $|X_i-X_j|>2b$, where b is the disk radius. van Kampen [22] and Lebowitz and Penrose [23] proved that the pressure Π of a fluid with hard-particle repulsion and long-range attractive forces satisfies the van der Waals-like equation of state

$$\Pi=\Pi^{\mathrm{hp}}+\tfrac{1}{2}\alpha\rho^2, \tag{30}$$

with

$$\alpha=\int_0^\infty U(R)2\pi R\,dR,$$

where Π^{hp} is the pressure of the hard-particle reference fluid with vanishing attractive interactions ($\rho=N/A$ is the particle density, k is the Boltzmann constant, and T is the temperature). Combining the relation $U(R)=-P^2G(R)$ and Eq. (23), the coefficient α is easily computed:

$$\alpha=-P^2\int_0^\infty G(R)2\pi R\,dR=-\left(\frac{P^2}{B}\right)d. \tag{31}$$

A different way to derive Eqs. (30) and (31) is given in Appendix C. Using an accurate equation of state for the hard core contribution Π^{hp} [24], Eq. (30) can be written as

$$\frac{\Pi a_0}{kT}=\frac{\psi}{(1-\psi)^2}-\psi^2\left(\frac{d}{D}\right), \tag{32}$$

where

$$D=\left(\frac{2a_0BkT}{P^2}\right) \tag{33}$$

is a characteristic stack thickness, and the dimensionless density $\psi=Na_0/A$ is equal to the fraction of substrate area covered by adsorbed particles ($a_0=\pi b^2$, the area of a disk). The equation of state (32) is related to the chemical potential

TABLE I. Interaction potentials of circularly symmetric particles and long parallel rods for large distances $R>\sqrt{\xi d}$ and $|x|>\sqrt{\xi d}$. (The rod length L is supposed to be much larger than the interaxial distance $|x|$.)

Supported bilayer with $\sigma\neq 0$
globes
$U(R)=-\frac{P_1P_2}{\sqrt{8\pi B'}\,\xi_I^2}\left(\frac{\xi_I}{R}\right)^{1/2}\exp\left(-\frac{R}{\xi_I}\right)$
parallel rods
$U(\lvert x\rvert)=-\frac{\hat{P}_1\hat{P}_2L}{2B'\xi_I}\exp\left(-\frac{\lvert x\rvert}{\xi_I}\right)$
Supported bilayer with $\sigma=0$
globes
$U(R)=-\frac{P_1P_2}{4\sqrt{\pi}B'\xi_{II}^2}\,h\left(\frac{R}{\xi_{II}\sqrt{2}}\right)\left(\frac{\xi_{II}}{R}\right)^{1/2}\exp\left(-\frac{R}{\xi_{II}\sqrt{2}}\right)$
parallel rods
$U(\lvert x\rvert)=-\frac{\hat{P}_1\hat{P}_2L}{2\sqrt{2}B'\xi_{II}}\left[\cos\left(\frac{\lvert x\rvert}{\xi_{II}\sqrt{2}}\right)+\sin\left(\frac{\lvert x\rvert}{\xi_{II}\sqrt{2}}\right)\right]\exp\left(-\frac{\lvert x\rvert}{\xi_{II}\sqrt{2}}\right)$
Stack of bilayers
globes
$U\left(r=\frac{R}{\sqrt{\xi d}}\right)=-\frac{(2\pi)^{1/4}P_1P_2}{\pi^2B\xi}\,h\left(\frac{1}{2}\sqrt{\pi}r\right)\left(\frac{1}{r}\right)^{1/2}\exp\left(-\frac{1}{2}\sqrt{\pi}r\right)$
parallel rods
$U\left(r_1=\frac{\lvert x\rvert}{\sqrt{\xi d}}\right)=-\frac{2\hat{P}_1\hat{P}_2L}{\pi^{3/2}B}\left(\frac{d}{\xi}\right)^{1/2}\left[\cos\left(\frac{1}{2}\sqrt{\pi}r_1\right)+\sin\left(\frac{1}{2}\sqrt{\pi}r_1\right)\right]\exp\left(-\frac{1}{2}\sqrt{\pi}r_1\right)$

$$\mu=kT\left[\ln\frac{\psi}{1-\psi}+\frac{(1+\psi-\psi^2)}{(1-\psi)^2}-2\left(\frac{d}{D}\right)\psi\right] \qquad (34)$$

IV. DISCUSSION

The effective pair potentials resulting from the substrate-mediated interaction for circularly symmetric particles (‘‘globes’’) and parallel rods are summarized in Table I. These results, based on the linear theory of elasticity, are restricted to the case of small film deformations. Furthermore, for very thick stacks the substrate-induced interaction is modified by thermal fluctuations, which are not incorporated into the present approach. Apart from oscillations for large distances R the potentials decay exponentially. If interacting particles are of the same type $(P_1=P_2=P)$, the force between them is always attractive for small and moderate distances (Fig. 2). The sign of P is positive if the film thickness is enlarged $(u>0)$, just below an adsorbed convex particle, and negative if the thickness is diminished. Different particles with opposite sign of P repel each other. The interaction becomes stronger when the compression-expansion modulus decreases or the decay length for elastic deformations increases. However, there is a restriction in material optimizations for bilayers, since the characteristic decay length ξ_{II} for membrane thickness perturbations is expected to be comparable to the bilayer thickness d_0 [1]. Similarly, in the case of large surface tensions σ' the corresponding characteristic length $\xi_I=\sqrt{\sigma'/B'}$ should hardly exceed the bilayer thickness. The characteristic length $\xi=\sqrt{K/B}$ for lamellar phases is comparable to the distance between adjacent lamellae [20]. In contrast to a bilayer, however, multilayer stacks offer the additional possibility to modify the interaction radius of adsorbed colloidal molecules by changing the stack thickness d. According to the results summarized in Table I the pair potentials decay exponentially with a decay length proportional to $\sqrt{\xi d}$. It should be mentioned that the interaction range of inclusions embedded within a smectic film also depends on the square root of the film thickness [11]. Obviously, utilizing the Langmuir-Blodgett technique, the interaction radius can be controlled by varying the number of bilayers in the stack. If $d\gg d_0$, the theory for adsorbed molecules is also applicable to integral membrane proteins which span the upper bilayer of the film (Fig. 1). In this case the parameters P_i $(i=1$ and $2)$ are phenomenological constants which cannot be identified with the force defined by Eq. (29).

If the stack is moderately thick $(\sqrt{\xi d}\gg b)$ the range of the substrate-mediated interaction is much larger than the hard core radius b of the adsorbed molecules. In this case the van der Waals equation (32) is applicable. The adsorbate aggregates if the attractive forces are sufficiently strong or if the interaction radius is sufficiently large. In the present case the

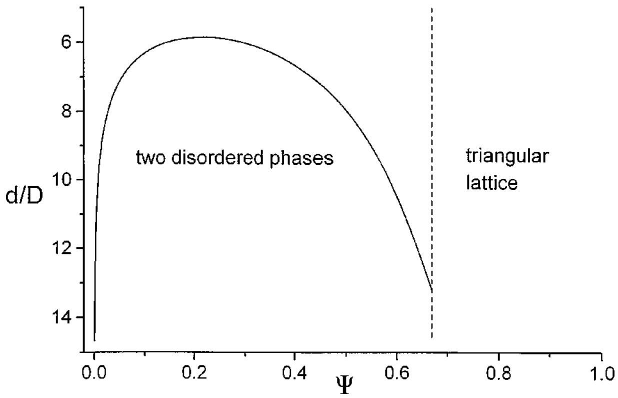

FIG. 3. In the phase diagram the reduced stack thickness is plotted vs the density. If the thickness exceeds the critical value $d_C = 5.8D$, an equilibrium of gaslike and liquidlike states is stable for moderate densities. For high densities hard disklike particles form a two-dimensional crystal.

increase of the interaction radius can induce the aggregation. Using Eqs. (32) and (34), the phase diagram in Fig. 3 is evaluated by applying the conditions $\Pi(\psi')=\Pi(\psi'')$ and $\mu(\psi')=\mu(\psi'')$ for mechanical and chemical equilibria, respectively. The transition from a gaslike to a fluidlike state occurs if the stack thickness d exceeds the critical value $d_C = 5.8D$. Attractive forces have only little influence on the freezing density of a hard disk fluid [25]. Crystalline order with a triangular lattice is always stable for sufficiently high densities ($\psi > 0.67$). Figure 3 suggests that the two-dimensional fluid disappears for $d/D > 14$, and the aggregation of the gaslike phase immediately leads to a triangular lattice.

A rough estimation of the magnitude of the characteristic thickness D [Eq. (33)] requires the force P exerted by a colloidal molecule. For peptides adsorbed on lipid bilayers, some information on P can be deduced from x-ray experiments [17]. These experiments revealed that peptides cause a thickness reduction of lipid bilayers. It was found for a bilayer surrounded by water ($\sigma \simeq 0$) that just adjacent to a peptide molecule the bilayer thickness is reduced by about 0.2 nm [17]. On the other hand, a thickness reduction $u(0) = PG_{II}^{S}(0)$ can be achieved by a point force with strength $P<0$. Using Eq. (12) and taking into account $\mathrm{Im}\,K_0(0) = -\pi/4$, we obtain $u(0) = -|P|/(8B'\xi_{II}^2) = -|P|/(8\sqrt{B'K'})$ in accord with the result of Boulbitch [19]. Since the decay length for thickness alterations ξ_{II} is comparable to the bilayer thickness [20], a possible choice is $\xi_{II} \simeq 1.2\,\mathrm{nm}$ [17]. Choosing the value $B' \simeq 1.7\times 10^{-11}\,\mathrm{N/nm^3}$ [18] for a dimyristoylphosphatidylcholine (DMPC) bilayer and $u(0) \simeq -0.2\,\mathrm{nm}$, the force $P = 8u(0)B'\xi_{II}^2 \simeq -3.9\times 10^{-11}\,\mathrm{N}$ is estimated. We assume that a peptide exerts the same normal force P onto a bilayer stack. The approximate formula $B = B'd_0$ yields the reasonable value $B = 5.1\times 10^{-11}\,\mathrm{N/nm^2}$ for the compression modulus of a lamellar phase. Finally, for $a_0 \simeq 3\,\mathrm{nm}^2$ (the cross section of an adsorbed peptide [17]) and the temperature $T \simeq 300\,\mathrm{K}$ the critical thickness for aggregation $d_C = 5.8D \simeq 14\,\mathrm{nm}$ is found. In this case a film containing five bilayers would be sufficient to observe a phase transition from a gaslike to a fluidlike state.

Membrane-mediated attractive forces can occur also between long molecules such as DNA. Recently Fang and Yang [6] investigated DNA adsorbed on a supported lipid bilayer. Utilizing atomic force microscopy, they found ordered domains of parallel DNA molecules with regular interaxial spacing of about 5 nm, which is considerably lower than the length of the DNA. It was proposed that this distance results from the balance of repulsive electrostatic forces and membrane-mediated attractive forces [6,7]. The potentials summarized in Table I suggest that the attraction force for parallel rods could be rather strong, since $U(|x|)$ is always proportional to the rodlength L. However, further experiments are necessary to explain the stability of these DNA arrays.

In conclusion, the range of membrane-mediated forces between adsorbed colloidal molecules can be controlled by changing the number of layers in a Langmuir-Blodgett film. The interaction radius is proportional to the square root of the stack thickness. This effect can lead to the aggregation of the adsorbate if the number of layers exceeds a critical value.

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APPENDIX A

The integral

$$G(x-\overline{x},y-\overline{y},z)=\frac{1}{(2\pi)^2}\int\int\frac{\sinh[q^2\xi(d+z)]\exp[iq_x(x-\overline{x})+iq_y(y-\overline{y})]}{q^2[\xi B\cosh(q^2\xi d)+\sigma\sinh(q^2\xi d)]}\,dq_y dq_z$$

is transformed by introducing polar coordinates $(q=\sqrt{q_x^2+q_y^2},\phi)$. The integration over the polar angle ϕ results in

$$G(R,z)=\frac{1}{2\pi}\int_0^\infty\frac{\sinh[q^2\xi(d+z)]J_0(qR)}{q^2[\xi B\cosh(q^2\xi d)+\sigma\sinh(q^2\xi d)]}\,q\,dq,$$

where $R=\sqrt{(x-\overline{x})^2+(y-\overline{y})^2}$, and $J_0(qR)$ is the Bessel function of zero order. This function can be replaced by the Hankel function of zero order by applying Eq. (8). Then the integral is replaced by

$$G(R,z)=\frac{1}{4\pi}\int_{-\infty}^{+\infty}\frac{P(q,z)}{Q(q)}\,dq,$$

where

$$P(q,z)=\frac{\sinh[q^2\xi(d+z)]}{q}H_0^{(1)}(qR)$$

and

$$Q(q)=\xi B\cosh(q^2\xi d)+\sigma\sinh(q^2\xi d).$$

Applying the method of residues, we obtain

$$G(R,z)=2\pi i\sum_m\frac{P(q_m,z)}{4\pi Q'(q_m)},$$

where $Q'=dQ/dq$, and the complex numbers q_m localize the poles in the upper half of the complex plane. The poles are evaluated from the equation $Q(q)=0$, which has an infinite number of solutions. The squares q_m^2 obey the equation

$$q_m^2\xi d=\left(m+\frac{1}{2}\right)\pi i-\frac{1}{2}\ln\left[\frac{1+\sigma/(\xi B)}{1-\sigma/(\xi B)}\right],$$

where m is an integer number. If the condition $\sigma/(\xi B)\ll 1$ is satisfied, the logarithmic term can be neglected, and then the zeros of $Q(q)$, with $\operatorname{Im}q>0$, are

$$q_n^+=i\beta\left(\frac{(n+\frac{1}{2})\pi}{\xi d}\right)^{1/2}\quad\text{and}\quad q_n^-=i\beta^*\left(\frac{(n+\frac{1}{2})\pi}{\xi d}\right)^{1/2},$$

where $n=0,1,2,3,\ldots$, $\beta=(1+i)/\sqrt{2}$, and β^* is the complex conjugate of β. Thus we obtain

$$G(R,z)=2\pi i\sum_{n=0}^{\infty}\left[\frac{P(q_n^+,z)}{4\pi Q'(q_n^+)}+\frac{P(q_n^-,z)}{4\pi Q'(q_n^-)}\right].$$

Inserting the relations for q_n^+ and q_n^-, the Green's function can be cast into the form

$$G(R,z)=\frac{i}{4B\xi^2 d}\sum_{n=0}^{\infty}(-1)^n\sin\left[\frac{\pi(\frac{1}{2}+n)(d+z)}{d}\right]\times\left[\frac{H_0^{(1)}(q_n^+R)}{(q_n^+)^2}+\frac{H_0^{(1)}(q_n^-R)}{(q_n^-)^2}\right].$$

Finally, taking into account the relation between the Hankel function and the modified Bessel function of the first kind [16], namely, $H_0^{(1)}(q_n^+R)=(2/\pi i)K_0(q_n^+R/i)$, and the corresponding formula for $H_0^{(1)}(q_n^-R)$, we arrive at

$$G(R,z)=-\frac{1}{\pi^2B\xi}\sum_{n=0}^{\infty}\frac{1}{n+\frac{1}{2}}\times\cos\left[\frac{\pi z}{d}\left(n+\frac{1}{2}\right)\right]\operatorname{Im}K_0\left(\frac{\beta\sqrt{(n+\frac{1}{2})\pi}R}{\sqrt{\xi d}}\right).$$

In a similar way, the Green's function for rods

$$G(|x|,z)=\frac{2}{(2\pi)^{3/2}B}\left(\frac{d}{\xi}\right)^{1/2}\sum_{n=0}^{\infty}\frac{g_n(|x|)}{\left(n+\frac{1}{2}\right)^{3/2}}\times\cos\left[\frac{\pi z}{d}\left(n+\frac{1}{2}\right)\right]\exp\left[-\frac{\sqrt{(1+\frac{1}{2})\pi}|x|}{\sqrt{2\xi d}}\right],$$

with

$$g_n(|x|)=\cos\left(\frac{\sqrt{(n+\frac{1}{2})\pi}|x|}{\sqrt{2\xi d}}\right)+\sin\left(\frac{\sqrt{(n+\frac{1}{2})\pi}|x|}{\sqrt{2\xi d}}\right)$$

is obtained.

APPENDIX B

We consider an infinitely thick stack of layers filling the half-space $-\infty<z\leqslant 0$. In the case of long rods, the expression

$$G(x-\bar{x},z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(q^2\xi z+iq(x-\bar{x}))}{q^2(\xi B+\sigma)}dq$$

satisfies the required boundary condition

$$B\left(\frac{\partial G}{\partial z}\right)_{z=0}-\sigma\Delta G(z=0)=\delta(x-\bar{x}).$$

Integration yields, straightforwardly,

$$G(x-\bar{x},z)=\frac{1}{2(\xi B+\sigma)}\left[|x-\bar{x}|\operatorname{erf}\left(\frac{x-\bar{x}}{\sqrt{\xi|z|}}\right)+\left(\frac{\xi|z|}{\pi}\right)^{1/2}\times\exp\left(-\frac{(x-\bar{x})^2}{4\xi|z|}\right)\right].$$

For $z=0$ this result is simplified to

$$G(x-\bar{x})=\frac{|x-\bar{x}|}{2(\xi B+\sigma)}.$$

APPENDIX C

We consider an ensemble of N globular hard particles attached to a surface. If the attractive forces have a large interaction radius, the free energy can be expressed as the sum $F^{\mathrm{hs}}+F$, where F^{hs} is the free energy of the hard-particle reference system, and the second term F is allocated to the attractive interaction [26]. Attractive substrate-mediated forces result from the free energy contribution

$$F=-\tfrac{1}{2}\int G(|X-\bar{X}|)f(X)f(\bar{X})d^2X\,d^2\bar{X},$$

where $f(X)=\Sigma_{i=1}^{N}P(X-X_i)$. Using the relations

$$f(X)=\int P(X-\bar{\bar{X}})\sum_{i=1}^{N}\delta(\bar{\bar{X}}-X_i)d^2\bar{\bar{X}},$$

$$f(\overline{X}) = \int P(\overline{X} - \overline{\overline{\overline{X}}}) \sum_{i=1}^{N} \delta(\overline{\overline{\overline{X}}} - X_i) d^2\overline{\overline{\overline{X}}},$$

the elastic free energy can be rewritten as

$$F = -\tfrac{1}{2} \int d^2\bar{\bar{X}} d^2\overline{\overline{\overline{X}}} \hat{G}(|\bar{\bar{X}} - \overline{\overline{\overline{X}}}|) \sum_{i,j} \delta(\bar{\bar{X}} - X_i) \delta(\overline{\overline{\overline{X}}} - X_j),$$

with

$$\hat{G}(|\bar{\bar{X}} - \overline{\overline{\overline{X}}}|) = \int d^2X\, d^2\overline{X} P(X - \bar{\bar{X}}) G(|X - \overline{X}|) P(\overline{X} - \overline{\overline{\overline{X}}}).$$

Averaging over a statistical ensemble, we obtain

$$F = -\tfrac{1}{2} \int \hat{G}(|X - \overline{X}|) \langle \rho(X) \rho(\overline{X}) \rangle d^2X d^2\overline{X},$$

where

$$\langle \rho(X) \rho(\overline{X}) \rangle = \left\langle \sum_{i,j} \delta(X - X_i) \delta(\overline{X} - X_j) \right\rangle$$

is the density correlation function. For the limit $|X - \overline{X}| \to \infty$, or more precisely, if $|X - \overline{X}|$ is considerably larger than the hard core radius of the particles, the pair correlation function can be written as $\langle \rho(X) \rho(\overline{X}) \rangle = \langle \rho(X) \rangle \langle \rho(\overline{X}) \rangle = \rho^2$ if the system is homogeneous. Furthermore, for large values of $|X - \overline{X}|$, the replacement $\hat{G}(|X - \overline{X}|) \to P^2 G(|X - \overline{X}|)$ is also justified. The resulting elastic free energy

$$F = -\frac{NP^2\rho}{2} \int_0^\infty G(R) 2\pi R\, dR$$

is accompanied by the pressure contribution

$$\Delta\Pi = \frac{\rho^2}{N} \partial F / \partial \rho = -\frac{P^2\rho^2}{2} \int_0^\infty G(R) 2\pi R\, dR,$$

in accord with the corresponding term in Eq. (30).

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